The significant parameters in defining the movement of the Beetle and its capture of intersection points have been ascertained and implemented within the code. The sample grid generated around several shapes shows the versatility of the code. It is intended to extend this methodology to shapes in three dimensions by converting the movement of the Beetle to that of a "paint brush" (defined by an elemental arc), which traverses an arbitrary surface (paints the surface), and thereby collating the intersecting boundaries of the hexahedral cells. A flow solver based on Colella et al.[1] is being developed to generate flow solutions.

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# Comparison of Deterministic and Stochastic Optimization Algorithms for Generic Wing Design Problems

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## Introduction

IN the past decade, a variety of numerical optimization algorithms have been extensively used to address multidisciplinary design optimization (MDO) problems that deal with design processes that are dependent on interactions among several engineering disciplines. MDO consists of many challenging features, such as a heterogeneous mix of analysis codes for evaluating objective functions, a large number of design variables, discrete design parameter values, and complex constraints. Local optimization strategies, such as gradient-based algorithms, have been widely applied to engineering system designs.[1,2] Although the number of design iterations required by local optimizers can be small, a major shortcoming of local optimizers is that they may get trapped in local optima. Although the performance of local optimizers can be enhanced by sensitivity analysis, such as that given in Ref. 2, which reduces the number of objective function evaluations in the optimization process, it is often accomplished at the expense of additional work in constructing models for estimating sensitivities. In some cases the use of approximate derivatives in estimating sensitivities (see Ref. 1) can lead to a loss of accuracy in the optimization process.

Besides deterministic methods for optimization, global optimization methods that are robust in avoiding local optima have found applications recently in practical designs. The simulated annealing (SA) method and the genetic algorithm (GA) have recently shown promise in addressing MDO engineering problems. Both SA algorithms and GAs are stochastic in nature and are easily implemented in robust computer codes compared with deterministic methods. However, the SA method and the GA require a large number of function evaluations and longer computation time, especially in complex design problems that couple interactions between multiple disciplines, such as fluids and structures, and that have a large number of design variables. Hence, to shorten the computation time, it is imperative that SA and the GA be implemented by high-performance computing technologies such as parallel computing.

Our aim is to compare the performance of deterministic and stochastic optimization methods by applying these to generic aircraft wing design problems to guide the use of these methods for MDO problems. The design problem considered for this study concerns the design of the optimal wing shape for minimizing drag with weight of the wing as a constraint so that the interaction between aerodynamics and structural weight influences the determination of the best wing shape. The optimization algorithms used for this problem are the SA method, the GA, the gradient-based method (GM), the Powell search method, the parallel SA (PSA) method, and the parallel GA (PGA). These methods are outlined briefly in the following section, and their performances in design optimization of the selected problem are compared.

## Optimization Algorithms

The optimization algorithms used in this study are classified into stochastic (global search) methods (SA, GA, etc.) and deterministic (local search) methods (GM, Powell method, etc.). SA is a search of the design space with the goal of finding a global minima, as in Ref. 3. In SA, the optimization problem is simulated as an annealing process, and SA possesses a formal proof of convergence to global optima, although this proof relies on a very slow cooling schedule and sufficiently large initial temperature. The GA is a computerized search, and optimization algorithms are based on the mechanics of natural genetics and natural selection, as Ref. 4. The basic mechanism of GAs is provided by reproduction and crossover processes. Hence the GA can be applied for optimizing objective functions in design spaces that are multimodal or discontinuous. The GA searches from a population of points, and the survival-of-the-fittest strategy increases the probability of finding the global optimum in multimodal or convex search spaces.

The GMs have the advantage of fast convergence in locating local minima, for which reasonably accurate derivatives can be estimated in a cost-effective manner. However, the tendency for the GM's getting trapped in local minima is high. GMs are suitable for searching convex design spaces with continuous derivatives. In this work, the Broydon–Fletcher–Goldfarb–Shanno (BFGS) method outlined in Ref. 5 is used for the optimization process. If the objective function does not possess continuous derivatives, a direct search method such as Powell's method, as outlined in Ref. 6, might be more suitable for design optimization. Powell's method uses a history of previous solutions to create new search directions and only function values at different points are needed.

It is imperative that methods to reduce computational time be used for implementing global optimization algorithms. A number of possibilities exist for this. One way is to incorporate modifications to basic SA and the GA; for example, in SA, a wise choice of the cooling scheme and the length of Markov's chains can result in moderate computational savings. The advent of parallel-processing architecture and efficient message-passing libraries offer the possibility of parallelizing SA and the GA. A parallel SA algorithm proposed by Diekmann et al.[7] was developed with the MPI library described in Ref. 8 with the MPT tool and applied to the same problem. The distributed parallel GA method shows that migration introduced between multiple processors and set in a loop is an efficient way for

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doing the same problem on coarse-grained multiple processors, as described in Refs. 9 and 10.

## Statement of the Generic Wing Design Problem

The design problem used to compare the various optimization algorithms concerns the design of a wing shape such that the aerodynamic efficiency of the wing reaches a maximum value during cruise with the wing weight acting as a constraint. In cruise flight the Breguet–Range equation is given by[11]

$$R = (V/f)(L/D)\,\ln(W_i/W_{i+1}) \tag{1}$$

where $R$ is the range, $f$ is the specific fuel consumption, $V$ is the flight velocity, $L/D$ is the lift-to-drag ratio or aerodynamic efficiency, and $W_i$ and $W_{i+1}$ are the weights at the start and the end of the cruise flight, respectively. For maximizing range it is imperative that the aerodynamic efficiency be maximized. The goal then is, by use of optimization methods outlined in the preceding section, to determine the shape of the wing for minimizing $D/L$ (or maximizing $L/D$) with the wing weight as a constraint. The $D/L$ ratio can be formulated in detail by the analytic formula for aerodynamic analysis as given by Raymer[11]:

$$L = C_L q S, \qquad C_L = C_{L\alpha}\alpha \tag{2}$$

$$C_{L\alpha} = 2\pi A_R \Big/ \left[2 + \sqrt{4 + (A_R\beta/\eta)^2(1 + \tan^2\lambda/\beta^2)}\right] \tag{3}$$

$$C_{Di} = C_L^2 \Big/ (\pi A_R e), \qquad C_{D0} = C_f F Q \tag{4}$$

$$e = 4.61\left(1 - 0.045A_R^{0.68}\right)(\cos\lambda)^{0.15} - 3.1 \tag{5}$$

$$C_f = 0.455 \Big/ \left[(\log_{10} R_e)^{2.58}(1 + 0.144M^2)^{0.65}\right] \tag{6}$$

$$F = \left\{1 + [0.6/(x/c)_m](t/c) + 100(t/c)^4\right\}[1.34M^{0.18}(\cos\lambda)^{0.28}] \tag{7}$$

$$C_D = C_{Di} + C_{D0}, \qquad D = C_D q S, \qquad D/L = C_D/C_L \tag{8}$$

$$W_{\text{wing}} = 0.0106(W_{\text{dg}}N_z)^{0.5}S^{0.622}A_R^{0.75}(t/c)^{-0.4}(\cos\lambda)^{-1} \tag{9}$$

where $W_{\text{dg}}$ is the design gross weight (in pounds), $N_z$ is the ultimate load factor, $Q$ is the interference-effect factor on drag, $t/c$ is the airfoil thickness-to-chord ratio, $c$ is the mean aerodynamic chord, $b$ is the wing span, $S$ is the wing reference area ($=bc$, ft$^2$) $A_R$ is the aspect ratio ($b^2/S$), $\beta = 1 - M^2$, $\eta = 0.95$–$1.0$, $Q = 1.0$, and $x/c_m$ is the chordwise location of the airfoil maximum thickness point; here 0.3 is adopted for the subsonic airfoils. Details of estimating other parameters such as $R_e$ and other parameters can be found in Ref. 11.

In the design optimization, the objective function is $D/L$. An external penalty function method is used to solve constraints optimization and can be presented as

$$\text{minimize } F(X), \qquad F(X) = D/L + \beta\,\max\left[0, \sum g_j^2\right] \tag{10}$$

where $X$ is the vector of design variables, that is, $X = (x_1, x_2, \ldots, x_n)$, the design constraints $g_j(X) \le 0$ are represented as inequality constraints, and $\beta$ is the coefficient of the penalty function. The four design variables for the generic problem considered are angle of attack $\alpha$, wing sweep angle $\lambda$, wing span $b$ (in feet), and mean aerodynamic chord $c$ (in feet). The design optimization is subject to six constraints, defined as follows:

$$\begin{aligned} &1.0\text{ deg} \le \alpha \le 10.0\text{ deg}, && 10.0 \le b \le 50.0 \\ &3.5 \le c \le 10.0, && 0.0\text{ deg} \le \lambda \le 35.0\text{ deg} \\ &0.5 \le A_R \le 15.0, && W_{\text{wing}} \le 2473\text{ (lb)}. \end{aligned} \tag{11}$$

## Results and Discussions

For enabling a comparative study, the flight Mach number $M = 0.7$ and $t/c = 0.12$ were used in initiating the optimization schemes, and the same termination criterion $|F(X_{n+1}) - F(X_n)| \le 0.001$ was used to terminate the optimization methods implemented, except for the GA, for which a reasonable number of generations were used. Table 1 compares the values of the optimum values of the objective function and design variables attained and the number of evaluations of the objective function ($N_o$) when the different optimization methods are used. In the application of SA algorithm, the initial temperature was set to 50 and the cooling parameter $\gamma = 0.5$, which controls the factor by which the temperature is reduced for

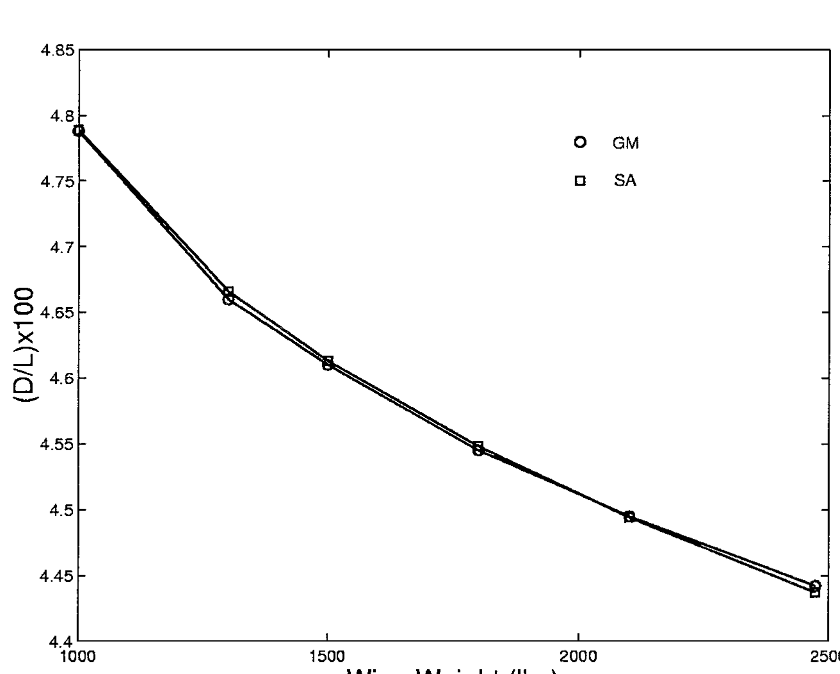


**Fig. 1 Optimized objective function vs wing weight.**

each iteration. A reasonable value of 10 was chosen for the length of the Markov chain. More than 600 objective function evaluations were required for satisfying the specified convergence criterion. In the application of the GA, 226 generations and 5 populations were chosen. The crossover parameter was 0.5, and the mutation parameter was 0.02. The accuracy was provided by the search interval divided by possibility parameter $P_o$ which was set at $2^8 - 1$. It can be seen that the GA takes more iterations than SA to reach nearly the same minimum. The deterministic GM based on the BFGS method reached convergence in 90 evaluations of objective function, making it the fastest. The Powell method also demonstrated fast convergence in searching the local maximum without requiring derivative information of the objective function. Deterministic algorithms takes fewer function evaluations to reach convergence compared with stochastic optimization methods. A parallel implementation of SA, as outlined, was also used in the design optimization. All parameters were kept the same as those used in the serial SA code. The MPT 1.3 (MPI library) compiler is applied on eight processors of an SGI Origin 2000 shared-memory computer. Optimized results were obtained after 148 function evaluations, and a satisfactory speedup of 4.4 (defined by the ratio of the evaluations on one processor to the evaluations on eight processors) was also achieved. A parallel implementation of the GA, i.e., the PGA, was used based on the same set of parameters used for the serial GA with each processor handling approximately five populations. On eight processors, the objective function reached a value of 4.443 after 395 evaluations of the objective function, and the speedup was 2.53. The variation of the objective function and the four design variables as a function of the wing weight optimized by the SA method and the GM are compared in Figs. 1 and 2 to show the differences between stochastic and deterministic optimization methods.

**Table 1 Optimal design results of the wing**

| Data | SA | GA | GM | Powell | PSA | PGA |
|---|---|---|---|---|---|---|
| $(D/L)10^2$ | 4.437 | 4.442 | 4.437 | 4.437 | 4.437 | 4.443 |
| $\alpha$ | 3.669 | 3.682 | 3.683 | 3.678 | 3.676 | 3.682 |
| $b$ | 45.59 | 44.99 | 45.56 | 45.50 | 45.51 | 44.99 |
| $c$ | 6.196 | 6.049 | 6.137 | 6.054 | 6.087 | 5.896 |
| $\lambda$ | 0.0370 | 1.102 | 0.344 | 0.104 | 0.156 | 1.102 |
| $N_o$ | 653 | 1000 | 90 | 152 | 148 | 395 |

## Conclusion

For the generic aircraft wing design problem considered here, it appears that deterministic optimization methods have the merits of fast convergence and fewer function evaluations compared with stochastic optimization methods. Deterministic methods are suitable for simple problems that have convex search domains. The stochastic optimization methods are general methods for handling discrete variables, nonconvex, and multimodel problems. PSA and PGA are emerging as promising tools for obtaining optimum designs and are efficient in reducing the computational efforts of stochastic methods. They are worth investigating further for integrating modern computational methods such as computational fluid dynamics and structural dynamics with computer-aided design to address complex MDO problems.

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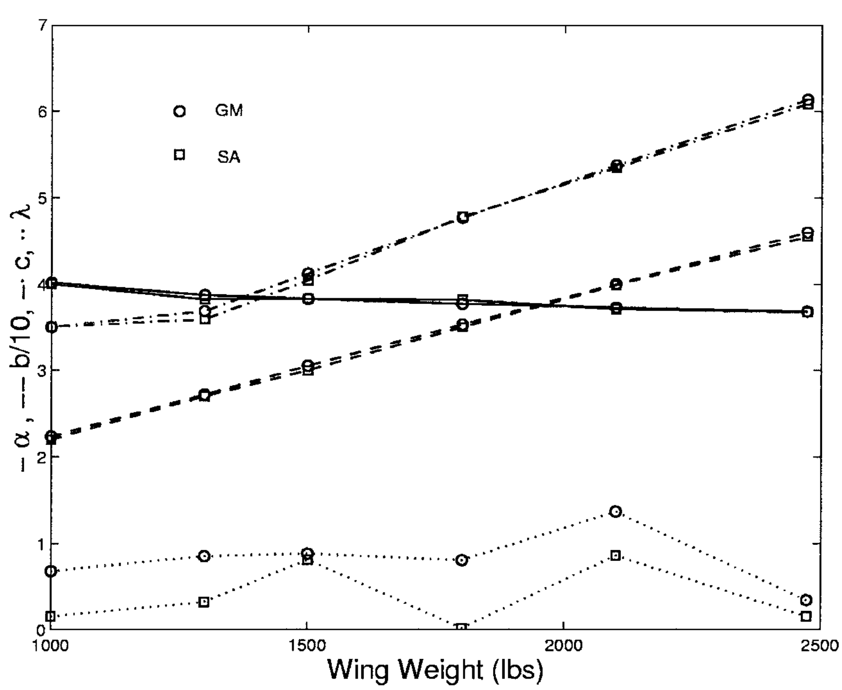


**Fig. 2 Design variables vs wing weight.**

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# Optimal Trajectory for a Minimum Fuel Turn

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## Nomenclature

| | | |
|---|---|---|
| $b$ | = | fuel burn |
| $\boldsymbol{c}_{\max}$ | = | vector of upper bounds |
| $\boldsymbol{c}_{\min}$ | = | vector of lower bounds |
| $D$ | = | drag |
| $g$ | = | gravity acceleration |
| $\boldsymbol{g}_{\max}$ | = | vector of upper bounds on algebraic constraints |
| $\boldsymbol{g}_{\min}$ | = | vector of lower bounds on algebraic constraints |
| $h$ | = | altitude |
| $L$ | = | lift |
| $m$ | = | aircraft total mass |
| $m_f$ | = | fuel mass |
| $T$ | = | engine thrust |
| $\boldsymbol{u}$ | = | assembled control vector |
| $\boldsymbol{x}$ | = | assembled state vector |
| $x_E$ | = | distance traveled east |
| $\boldsymbol{y}$ | = | vector of discretized state and control variables |
| $y_E$ | = | distance traveled north |
| $\alpha$ | = | angle of attack |
| $\gamma$ | = | flight path angle |
| $\delta_T$ | = | power level angle |
| $\epsilon$ | = | thrust angle with respect to body fixed axis |
| $\phi$ | = | bank angle |
| $\psi$ | = | heading |

## Introduction

FINDING optimal aircraft trajectories by using numerical optimization methods is a well-established area of research. Current methods in widespread use are based on discretization by using Hermite-Simpson collocation and direct solution using nonlinear programming methods. This type of method was pioneered by Hargraves and Paris[1] and is now perhaps the most widely used approach. In some cases, the numerically computed trajectories have also been verified by flight testing.[2]

In most cases numerical methods are demonstrated on problems in two dimensions, namely the vertical plane involving only longitudinal motion. However, many problems are truly three-dimensional and require a more general approach. Performance data models for aircraft are usually rather simple, in particular, the aerodynamic model. There is usually no aerodynamic data for sideslip or unsteady effects. If some restrictions are imposed on the three-dimensional motion of the aircraft, it is possible to solve problems in three dimensions by using only the standard performance data for the aircraft of interest.

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The purpose of this Note is to present a point-mass model suitable for solving multistage trajectory optimization problems in three dimensions. The model is first discussed, and the numerical method is described. Finally, the presented method is used to solve an interesting test problem related to in-flight flutter testing.

## Performance Model

The equations of motion for the aircraft are obtained by reducing the full six-degrees-of-freedom dynamic model of the aircraft described by Etkin,[3] assuming a point-mass model of the aircraft. Sideslip and unsteady aerodynamic effects are neglected because these effects have very little influence on the type of maneuvering considered here. Consequently, all maneuvering is assumed to take place without sideslip. The resulting equations of motion are given by the system of ordinary differential equations

$$m\dot{V} = T\cos(\alpha + \epsilon) - D - mg\sin\gamma \tag{1}$$

$$mV\dot{\gamma} = T\sin(\alpha + \epsilon)\cos\phi + L\cos\phi - mg\cos\gamma \tag{2}$$

$$mV\dot{\psi}\cos\gamma = T\sin(\alpha + \epsilon)\sin\phi + L\sin\phi \tag{3}$$

$$\dot{h} = V\sin\gamma \tag{4}$$

$$\dot{m}_f = -b \tag{5}$$

$$\dot{x}_E = V\cos\gamma\cos\psi \tag{6}$$

$$\dot{y}_E = V\cos\gamma\sin\psi \tag{7}$$

where a flat, nonrotating earth approximation is assumed.

## Multistage Trajectory Optimization

The equations of motion defined by Eqs. (1–5) can be rewritten in brief form as

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \tag{8}$$

The distances $x_E$ and $y_E$ are defined as algebraic constraints by integrating Eqs. (6–7), giving

$$x_E(t_F) = \int_{t=0}^{t_F} V(t)\cos\gamma(t)\cos\psi(t)\,\mathrm{d}t \tag{9}$$

and

$$y_E(t_F) = \int_{t=0}^{t_F} V(t)\cos\gamma(t)\sin\psi(t)\,\mathrm{d}t \tag{10}$$

Consequently, Eqs. (6) and (7) are not included in Eq. (8).

Additional requirements, such as restrictions on load factor $n_z$, dynamic pressure, lift coefficient, Mach number, and indicated airspeed $V_i$ are implemented as purely algebraic constraints in the form

$$\boldsymbol{g}_{\min} \le \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}) \le \boldsymbol{g}_{\max} \tag{11}$$

The differential Eq. (8) and algebraic Eq. (11) are discretized by using Hermite-Simpson collocation[4] with state variables interpolated as piecewise third-order polynomials and control variables as piecewise linear functions. The discretized state and control variables are stored in a finite-dimensional vector $\boldsymbol{y}$, together with the final time variables for each stage. The problem may be solved by using different stages, each involving a different set of state and algebraic equations representing different configurations of the aircraft or different vehicles, such as a combination of an aircraft and a missile. The multistage formulation used was developed in a previous study.[5]